$$\psi^2(x) = f^{-1}\{f(x) + 2\}$$
$$\psi^3(x) = f^{-1}\{f(x) + 3\}$$
$$\psi^n(x) = f^{-1}\{f(x) + n\}$$

Hence, amongst other consequences, it is evident that any functional equation of the form

$$A_n \psi^n(x) + A_{n-1}, \psi^{n-1}(x) + \ldots + A\psi(x) + A_0 x = 0,$$

in which A_n, A_{n-1}, &c., are constants, may be reduced at once to a linear equation in finite differences with constant coefficients.

We might also invert the function $\psi(x)$ since

$$\psi^{-1}(x) = f^{-1}\{f(x) - 1\}$$

Dr. Graves stated that, in a continuation of the present paper he would lay before the Academy the results which he had obtained in discussing the symbol

$$e^{L\frac{d}{dx} + M\frac{d}{dy} + N\frac{d}{dz}}$$

in which L, M, N, are functions of x, y, and z; and which has the effect of changing x, y, and z respectively into certain functions of x, y, and z, whose form depends upon that of L, M, and N. One example of this kind has been already communicated to the Academy in a paper read by Dr. Graves on the 9th of June, 1851.

The Chair having been taken by the Rev. Dr. Lloyd,

The President communicated the results of four years' experience, at his own observatory, of the effects produced by the vicinity of a railroad.

"Amid the ever increasing requirements of improved accuracy which the progress of science is pressing on astronomical observers, it becomes important to avoid every possibility of error; to remove every cause that may, in the slightest degree, add to the difficulties that inevitably oppose our advance

towards precision. The zenith sectors, the mural circles, the transits, which, forty years ago, were regarded as almost miracles of perfection, are fast sinking into neglect, and are displaced by inventions of higher promise. But instruments may be improved in vain unless they be used with skill proportioned to their improvement, and withdrawn from disturbances which may overwhelm their augmented power. Such disturbances, it is my purpose to show, may easily exist in the vicinity of a railroad. This matter has already attracted attention in consequence of two attempts to carry lines at about 800 feet from the Observatory of Greenwich, and experiments relating to it have been made by Captain Denham, Mr. Airy, and myself, whose results, differing only in degree, have been published by order of the House of Commons. Captain Denham, observing at Liverpool, on Sandstone Rock, took altitudes of the sun with a sextant, of low magnifying power, and a small reflecting surface of mercury, and found that the image was disturbed to the distance of 1110 feet. Immediately above the Tunnel he found the vibration scarcely sensible. Mr. Airy's experiments were made at Greenwich, on gravel, and at Kensal Green on clay; viewing with a small telescope the wires of a collimator reflected in mercury, his limits were in one case 1100 feet, in the other, 2200. My trials were made in the (now vanished) Dodder bank Distillery, 1055 feet from the Dublin and Kingstown line, observing with a large repeating circle the image of a land object reflected in a circular vessel of mercury, eight inches diameter. The soil here is mere silt, of great depth and *uniform density*, and, therefore, well adapted to propagate tremors. Accordingly, they were observed as far as the Terminus on one hand, 6434 feet along the line, and the Booterstown Station, on the other, 10893 feet. The wave of earth-vibration was much more extended before the engine than behind it, as, indeed, might be expected

from the nature of the disturbing force; it was also felt long before the sound of the train was audible.* I also found that when an object was bisected, and the level of the circle read, both were found to have varied after the passage of a few trains; and that the wires were ill-defined, and the stars blurred by the transit of a heavy engine.

"Sir James South, also, made reflection observations on stars, with a very powerful telescope, near Watford, on chalk full of fissures and very heterogenous, with results analogous to the preceding, but, I believe, of less intensity; these, unfortunately, have not yet been published.

"These observations, though fully significant, were made under conditions much less favourable than those which exist in fixed observatories; but I regret that I am able to complete them in this respect by my own experience. In 1848 the Ulster Railway was opened, and I soon found reason to congratulate myself on having resisted the original scheme which would have carried its line only 480 feet from my transit. The Armagh Observatory stands on ground probably corresponding in its vibrative power to that at Kensal Green. Underneath it is about 130 feet of dense clay, resting on mountain limestone, and the same clay is exposed in some deep cuttings on the line. The hill descends 90 feet, and rises from the valley 40 to the Terminus, which is the nearest point of the line, its horizontal distance being 2100 feet. The trains are light and few, five up and five down, and the velocity moderate, not averaging, so near the Terminus, 20 miles per hour.† The Terminus bears 50° west of my north meridian mark, and

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* This is adverse to an opinion expressed by Mr. Robert Stephenson, that this tremor of mercury is chiefly caused by the sonorous vibrations of the air. On one day the artillery was practising with shot from 24-pounders at the Pigeon House, and though the reports were heavy, the mercury was comparatively little affected.

† The average weight of engine and carriages is 40 ton, to which may be added 5 more for passengers.

the line crosses the meridian at an angle of 40° nearly, so that I scarcely anticipated any sensible tremors.

"I, however, have found that all the phenomena which I observed at Dodder bank are reproduced here.

"1. Disturbance of the mercurial horizon. The importance of the observations made with this can scarcely be overrated. In the case of stars, they give the index correction of a circle, the verification of its divisions, and the means of eliminating flexion; nor are they less useful for the transit. But another application of them, in which the reflected image of a telescope's own wires is made to coincide with the direct one, is perhaps even more valuable, and is coming into general use. Both require almost absolute quiescence of the mercury; an agitation of it, which no other method of examination can appreciate, will make the image nebulous and confused, or even cause it to oscillate round its true place. Even a moderate breeze of wind will do this; and therefore, as the amount of time during which such observations can be perfectly made is so limited, the introduction of any extra disturbance is injurious to an extent much more than proportioned to its actual duration. Now, I find that with an outgoing train, the image is so much disturbed as to make its coincidence doubtful for $4^m\ 36^s$ on an average: in one instance even for $6^m\ 17^s$, at which time, assuming the velocity as given above, the distance must have been 11,130 feet. The mercury vessel is the same which was used in my former experiments; but the magnifying power is the habitual observing one of 250. The time of disturbance for the incoming train is seven seconds less. During half this time the image cannot be seen, and occasionally the waves on the mercury are directly visible to the eye.

"2. In bisecting a star, or making a coincidence of the wire's images, the circle is moved by a fine screw. However perfect its centre work may be, or however complete the adaptation of counterpoises and friction wheels, an amount of fric-

tion must remain equal to many pounds at its place of action; all the parts, therefore, which intervene between the pivots of the instrument and the point of resistance of the largest screw must be under strain equal to the moment of this friction. But, as is well known, any slight tremor will relieve a part of the friction, and the circle must take a new position. Accordingly, I find that if (for example) I make the two images coincide by moving the reflected from right to left before a train, when that has passed it will be found to have started from two to four seconds towards the left, and *vice versâ*. This I think the most dangerous of all these disturbances; for if a train passes after the star is bisected, and while the observer is reading the microscope, the whole or a part of this jump will be changed to the star's place, and the direction of the error will depend on that in which the screw was last turned. I have, therefore, been obliged to establish it as a standing rule, that all such cases are to be noticed in the journal, and the observations rejected.

"3. The adjustments appear to be slightly affected; of this, however, I cannot speak with confidence, except in respect of the horizontality of the circle's axis, and its meridian position. The transit instrument has always been kept closely adjusted, and, therefore, cannot show these deviations; and the index correction of the circle is by its construction scarcely liable to change but by extreme violence. The circle's axis is levelled, by first correcting collimation in right ascension with two collimating telescopes, and then making the images of the central wire coincide; so that it can be at any time easily examined. It is kept in position by two check-nuts, on a strong screw, and formerly would remain many months before it erred to the amount of five seconds, which it was not allowed to pass. At present as great a change will sometimes occur in four days, and that abruptly. The disturbance in azimuth is less in amount, and I think independent of the other. I have not detected any effect on the clock.

"4. It certainly produces some optical indistinctness, which at so great a distance I did not expect. At first a train passed at noon; and we were surprised to find that the sun's limbs, though perfectly well defined at the first wires, would sometimes suddenly, as it were, shiver and become confused so as to preclude all observing; a little attention to the steam-whistle explained its cause, and had not the train been suppressed, we must have neglected solar observations for half the year. The effect was similar in both instruments, but I think greater in the circle. With respect to stars, it was chiefly studied on the pole-star, whose slow motion gave ample leisure to examine the appearances. In general, the star became a luminous blot of an elliptic form; but when the air was perfectly still, so that the definition was perfect, the phenomena were much more striking. In such cases the star is a mere point surrounded by a system of coloured rings; these, about five minutes before the train came in, elongated themselves with rapid oscillations nearly in the meridian. In another minute the central point began to dilate, and as it met the lesser axis of the distorted rings they coalesced, till all became a bright blur, as large as the inner ring, in two minutes more. As the speed was diminished, traces of the rings re-appeared, and when the train stopped, all was as at first. As respects stellar astronomy, this is not of much importance, for it will cause the loss of only two or at most three observations for each train, and only a rare chance could make it interfere with an occultation; with the sun it is otherwise, but I trust that there are no railway directors who would not in such a case alter their arrangements, so as to leave the 20^m before and after noon free.

"I will conclude by suggesting whether, henceforward, it will not be prudent, in selecting sites for observatories, to choose with special reference to the ground's capability of conducting tremors, and ascertain it by trial. From what I have stated, it seems probable that a deep and homogeneous substratum is in this respect the very worst; while the beautiful experiments

of Mr. Mallet, which have so lately been before the Academy, show that even the elasticity of granite is lowered by its joints and veins almost to a parity with incoherent sand. A gradual change of direction, such as is given by the little valley that I have described as intervening between me and the Terminus, has no effect; but I think that a more considerable one might exert considerable power in deadening the wave. It, therefore, is not unlikely that a mass of discontinuous rock, rising abruptly one or two hundred feet above a railway, would be but little affected by it.